A common stock pays dividends at the end of each year. Each dividend is 4% greater than the prior dividend and the effective rate of interest is 8%. Find the convexity of that common stock (the convexity measure sought is the one with respect to interest rate, not with respect to the force of interest).

A. 750 B. 900 C. 1000 D. 1250 E. 1750

Solution.
We know that the relationship of the price of the stock to the interest rate is

$$P = \frac{D_1}{i-g},$$

where D_1 is the dividend at the end of the first year, i is the interest rate, and g is the growth rate of the dividend. We see then that

$$\frac{d}{di}P = -\frac{D_1}{(i-g)^2},$$

and the duration of the stock is

$$-\frac{\frac{d}{di}P}{P} = -\frac{-\frac{D_1}{(i-g)^2}}{\frac{D_1}{i-g}} = \frac{1}{i-g}.$$

We also have

$$\frac{d^2}{di^2}P = \frac{d}{di}\left(-\frac{D_1}{(i-g)^2}\right) = -(-2D_1)\cdot\frac{1}{(i-g)^3} = \frac{2D_1}{(i-g)^3},$$

so that convexity equals

$$\frac{\frac{2D_1}{(i-g)^3}}{\frac{D_1}{i-g}} = \frac{2}{(i-g)^2}.$$

For the data given this equals

$$\frac{2}{(i-g)^2} = \frac{2}{(0.08-0.04)^2} = \frac{2}{0.04^2} = \frac{1}{0.0008} = 1250.$$

Answer D.