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Author of the ASM Manual (the “Been There Done That!” manual) for Course P/1: free excerpts and a practice exam available online <http://www.studymanuals.com/exam1.htm>

Course P/1 seminar: <http://www.math.ilstu.edu/actuary/prepcourses.html>

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Exercise for March 12, 2005

An urn contains 100 lottery tickets. There is one ticket that wins \$50, three tickets that win \$25, six tickets that win \$10, and 15 tickets that win \$3. The remaining tickets win nothing. Two tickets are chosen at random, with each ticket having the same probability of being chosen. Let X be the amount won by the one of the two tickets that gives the smaller amount won (if both tickets win the same amount, then X is equal to that amount). Find the expected value of X .

A. 0.1348 B. 0.0414 C. 0.2636 D. 0.7922 E. Does not exist

Solution.

The possible values of X are: 25, 10, 3, and 0. Furthermore,

$$\Pr(X = 25) = \frac{\binom{1}{1} \cdot \binom{3}{1} + \binom{3}{2}}{\binom{100}{2}} = \frac{3 + 3}{99 \cdot 50} = \frac{1}{33 \cdot 25} = \frac{2}{1650} = \frac{1}{825}$$

(one \$25 ticket and one \$50 ticket, or two \$25 tickets),

$$\Pr(X = 10) = \frac{\binom{4}{1} \cdot \binom{6}{1} + \binom{6}{2}}{\binom{100}{2}} = \frac{24 + 15}{99 \cdot 50} = \frac{13}{33 \cdot 50} = \frac{13}{1650}$$

(one \$10 ticket and one \$50 or \$25 ticket, or two \$10 tickets),

$$\Pr(X = 3) = 1 - \Pr(X = 10) - \Pr(X = 25) - \Pr(X = 0) =$$

$$= 1 - \frac{15}{1650} - \frac{\binom{25}{1} \cdot \binom{75}{1} + \binom{75}{2}}{\binom{100}{2}} = 1 - \frac{15}{1650} - \frac{62 \cdot 25}{1650} = \frac{85}{1650}$$

(for $X = 0$, one \$0 ticket and one other ticket, or two \$0 tickets),

$$\Pr(X=0) = \frac{\binom{25}{1} \cdot \binom{75}{1} + \binom{75}{2}}{\binom{100}{2}} = \frac{62 \cdot 25}{1650} = \frac{1550}{1650}.$$

Thus

$$E(X) = \frac{25 \cdot 2}{1650} + \frac{10 \cdot 13}{1650} + \frac{3 \cdot 85}{1650} = \frac{50 + 130 + 255}{1650} = \frac{435}{1650} = \frac{29}{110} \approx 0.2636.$$

Answer C.

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