Math 336
Spring 2007
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Solutions to HW # 11

11: 30. **Prove that an abelian group of order 2^n $(n \geq 1)$ must have an odd number of elements of order 2.**

PROOF. Let G be an abelian group of order 2^n. Let H denote the set of elements of order at most 2. Then, $H \leq G$ so H has even order. Since $e \in H$ is the only element of H that is not of order 2, G has an odd number of element of order 2.

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24: 16. **How many Sylow 5-subgroup of S_5 are there? Exhibit two.**

Solution: Recall $|S_5| = 2^3 \cdot 3 \cdot 5$. We know $n_5 \bmod 5 = 1$ and $n_5 \mid 24$. Hence, $n_5 = 1, 6$. Since $\langle (12345) \rangle$ and $\langle (13245) \rangle$ are two Sylow 5-subgroups, it follows that $n_5 = 6$.

24: 18. **Prove that a group of order 175 is abelian.**

PROOF. First, $175 = 5^2 \cdot 7$. Now, $n_5 \bmod 5 = 1$ and $n_5 \mid 7$. Hence, $n_5 = 1$. Next, $n_7 \bmod 7 = 1$ and $n_7 \mid 25$. Hence, $n_7 = 1$. Let H be a Sylow 5-subgroup and let K be a Sylow 7-subgroup. Since $|H| = 5^2$, it follows that H is abelian. Also, $|K| = 7$, so that K is abelian. Since $\gcd(|H|, |K|) = 1$, we have $H \cap K = \{e\}$. Further, since $n_5 = 1$ and $n_7 = 1$, both H and K are normal subgroups of G. Therefore, $HK \leq G$ and $H \cap K = \{e\}$ gives $|HK| = |H||K| = |G|$ so that $HK = G$. Since H and K are normal subgroups of G, $HK \approx H \oplus K$ so that $G \approx H \oplus K$. Since H and K are abelian, $H \oplus K$ is abelian and hence G is abelian. ■