Name: _____

Math 260, Spring 2007
H. Jordon

Exam 3

104 points possible. 100 points maximum.

1. (10 pts.) Find a minimum spanning tree using Prim's or Kruskal's algorithm in the weighted graph shown below. List or designate the edges in the order they are selected. Draw the resulting tree and find it's weight.

2. (10 pts.) Use Dijkstra's Algorithm to find a shortest path from vertex A to vertex M in the graph shown below. Label all the vertices as Dijkstra's Algorithm is applied, i.e., show all work.

3. (10pts.) Let G be a connected plane graph with $V \geq 2$ vertices, E edges, and R regions such that every vertex is on the exterior (unbounded) region. Prove that $E \leq 2V - 3$.

4. (15 pts.) Answer each of the following by CIRCLING True or False. No explanation necessary.

   (a) **True** or **False**: Every trail is a path.

   (b) **True** or **False**: In a hamiltonian graph, every edge belongs to a cycle.

   (c) **True** or **False**: Every eulerian graph contains a subgraph that is hamiltonian.

   (d) **True** or **False**: Every hamiltonian graph contains a subgraph that is eulerian.

   (e) **True** or **False**: A planar graph with chromatic number 3 must be hamiltonian.

5. (12 pts.) Give an example of each of the following or state that no such example exists.

   (a) A plane hamiltonian graph with chromatic number 2.

   (b) An eulerian and hamiltonian graph with chromatic number 2 that is not planar.

   (c) A eulerian graph that is not hamiltonian and not planar with chromatic number 3.

6. (15 pts.) Let G be the graph with $3n$ vertices $u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n, w_1, w_2, \ldots, w_n$ and all edges of the form $u_i v_j, v_i w_j, w_i u_j$ for $1 \leq i, j \leq n$.

   (a) For $n = 3$, draw the graph G.

   (b) Determine, with explanation, whether or not G is hamiltonian.

   (c) Determine, with explanation, whether or not G is eulerian.

7. (8 pts.) A tree has 33 vertices of degree 1, six of degree 3, and all other vertices of degree 7. Determine the number of vertices of degree 7.

8. (12 pts.) Suppose that in one particular semester there are students taking the following combinations of courses:

   - Math, English, French;
   - Math, English, Chemistry;
   - Biology, French, Math;
   - Biology, Physics, Math;
   - Chemistry, Physics, Math;
   - Computer Science, Math, Physics;
   - Computer Science, Math, Biology.

   We wish to determine the minimum number of examination periods for these courses.

   (a) Draw a graph that represents this situation.

   (b) Determine the chromatic number of this graph.

   (c) How is the chromatic number of this graph related to the minimum number of examination periods?

9. (12 pts.) The **complement** $\overline{G}$ of a graph G has $V(\overline{G}) = V(G)$ (so $\overline{G}$ and G have the same vertex set) and edge set $E(\overline{G}) = \{uv \mid uv \notin E(G)\}$ (so $\overline{G}$ has all the edges not in G).

(a) Find $\overline{C_7}$, the complement of a seven-cycle.

(b) Determine, with explanation, whether or not $\overline{C_7}$ is planar.