

Name: \_\_\_\_\_

Math 260, Fall 2007

H. Jordon

100 pts.

## Exam 2

105 points possible. 100 points maximum. **Show all work to receive full credit!**

1. (9 pts.) Leading zeros are not permitted in any part of this question. A digit is  $0, 1, 2, \dots, 9$ .

(a) How many 5-digit numbers are there (repetitions allowed)?

(b) How many 5-digit numbers are there if repetitions are not allowed?

(c) How many 5-digit numbers are there that have one or more repeated digit?

2. (10 pts.) Use the Euclidean Algorithm to find  $d = \gcd(2048, 10996)$ .

3. (6 pts.) Suppose  $G$  is a graph with 9 vertices and 27 edges. If  $G$  has 3 vertices of degree  $x - 1$ , 3 vertices of degree  $x$ , and 3 vertices of degree  $x + 1$ , what is  $x$ ?
4. (8 pts.) Find the prime factorization of 61589. Be sure to list all the numbers you tried as potential factors of 61589, even if they didn't work.
5. (10 pts.) Ryan has six weeks to prepare for the LSAT and not more than 60 hours available to study. He plans to study every day for at least an hour and for a whole number of hours every day. Show that no matter how he schedules his study time, there is a period of consecutive days during which he will have studied exactly 23 hours.

6. (12 pts.) Answer each of the following by circling TRUE or FALSE. No explanation necessary.

(a) **True** or **False**: There exists positive integers  $a$  and  $b$  such that  $a^2 = 3b^2$ .

(b) **True** or **False**: Every pseudograph is a graph.

(c) **True** or **False**: If  $\gcd(a, b) = 3$ , then for every  $n \in \mathbb{Z}$ , there exists integers  $x$  and  $y$  such that  $3n = ax + by$ .

(d) **True** or **False**: If  $a \mid bc$  for integers  $a, b, c$ , then  $a \mid b$  or  $a \mid c$ .

7. (6 pts.) Find the smallest positive integer  $a$  such that  $3^{80} \equiv a \pmod{13}$

8. (8 pts.) Convert  $(21012011)_3$  to base 5.

9. (12 pts.) Draw all nonisomorphic graphs with 4 vertices and 3 edges.

10. (12 pts.) How many integers between 1 and 5000 (inclusive) are divisible by 3 or 7 but not by 5?

11. (12 pts.) Use mathematical induction to prove that

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

for every natural number  $n$ . Be sure to identify the statement  $P(n)$ , state what you are assuming, and what you will prove. Clearly identify where you use the Inductive Hypothesis. Show all steps to receive full credit.