Name: ____________________

Math 260, Fall 2007
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100 pts.

Exam 1

Throughout this exam, $\mathbb{Z}$ denotes the set of integers and $\mathbb{N}$ denotes the set of positive integers. To receive full credit, you must show all work.

1. (12 pts.) Let $A = \{1, 2, 3, 4\}$. List the ordered pairs in a relation on A that is

   (a) reflexive and transitive but not symmetric

   (b) symmetric and transitive but not reflexive

   (c) reflexive and symmetric but not transitive.

2. (18 pts.) Let $A = \{\emptyset, a, \{a\}\}$. Answer each of the following by CIRCLING True or False.

   (a) **True** or **False**: $a \in A$.

   (b) **True** or **False**: $a \subseteq A$.

   (c) **True** or **False**: $\{a\} \in A$.

   (d) **True** or **False**: $\{a\} \subseteq A$.

   (e) **True** or **False**: $\emptyset \in A$.

   (f) **True** or **False**: $\emptyset \subseteq A$.

3. (10 pts.) Find the following sets if $A \setminus B = \{1, 5, 7, 8\}$, $B \setminus A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

(a) A

(b) B

(c) $A \cup B$

(d) $A \oplus B$

4. (16 pts.) Let A, B, C, and D be subsets of some universal set U. For each of the following statements, either prove the given statement is true or exhibit a counterexample to prove it is false (i.e., find **specific** examples of sets when the given statement is false).

(a) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

(b) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

5. (8 pts.) Prove that $(A \setminus B) \setminus C = A \setminus (B \cup C)$ for sets A, B, and C. Recall that Venn diagrams are not proofs; you may however use set theoretic identities.

6. (16 pts.) Define a relation $\mathcal{R}$ on $\mathbb{Z}$ by $(a, b) \in \mathcal{R}$ if $a + 5b$ is a multiple of 6.

   (a) Find three ordered pairs (a, b) with $(a, b) \in \mathcal{R}$ and three ordered pairs (a, b) with $(a, b) \notin \mathcal{R}$.

   (b) Prove that $\mathcal{R}$ is an equivalence relation on $\mathbb{Z}$.

   (c) Find the equivalence class $\overline{1}$.

7. (20 pts.) For each function below, determine whether or not f is **one-to-one** and **onto**. If f is one-to-one, **prove it** and be sure to state what you will assume and what you will show. If f is not one-to-one, give a **specific example** that shows it is not one-to-one and explain why your example works. If f does map onto its target, **prove it** and be sure to explain all steps. If f does not map onto its target, give a **specific example** that shows it doesn't map onto its target and explain.

   (a) $f : \mathbb{Z} \to \mathbb{Z}$ where $f(n) = \lfloor n/4 + 3 \rfloor$

   (b) $f : \mathbb{Z} \to \mathbb{Z}$ where $f(n) = 3n + 5$