

Let R be a set on which two operations \oplus and \odot are defined.

The following are “nice” properties we would like to hold for all $a, b, c \in R$:

1. $a \oplus b \in R$ (*closure of addition*)
2. $a \oplus (b \oplus c) = (a \oplus b) \oplus c$ (*addition is associative*)
3. $a \oplus b = b \oplus a$ (*addition is commutative*)
4. $\exists 0_R \in R$ such that $a \oplus 0_R = a = 0_R \oplus a \forall a \in R$
(*existence of additive identity*)
5. The equation $a \oplus x = 0_R$ has a solution in R for all $a \in R$.
(*existence of additive inverses*)
6. $a \odot b \in R$ (*closure of multiplication*)
7. $a \odot (b \odot c) = (a \odot b) \odot c$ (*multiplication is associative*)
8. $a \odot (b \oplus c) = a \odot b \oplus a \odot c$ and $(a \oplus b) \odot c = a \odot c \oplus b \odot c$
(*distributive laws*)
9. $a \odot b = b \odot a$ (*multiplication is commutative*)
10. $\exists 1_R \in R$ such that $a \odot 1_R = a = 1_R \odot a \forall a \in R$
(*existence of multiplicative identity*)
11. Whenever $a \odot b = 0_R$ in R then $a = 0_R$ or $b = 0_R$.
(*no divisors of zero*)
12. The equation $a \odot x = 1_R$ has a solution in R for all $a \neq 0_R \in R$.
(*existence of multiplicative inverses*)