

Study Guide for Exam 3

In what follows is a **brief** synopsis of what we have covered in Chapter 4. Use this list a guide to help you make up your own study guide.

On the exam, you can expect several proofs, TRUE/FALSE questions, and give-an-example-of type questions (be able to give an example of anything defined below). The problems that have been assigned in class (but not necessarily collected) or very similar problems could appear on the exam; therefore it is **highly** recommended that you make every effort to complete those problems.

Exam 3 Topics:

1. Polynomial rings, leading coefficient, degree of $f(x)$.
2. Addition and multiplication in $R[x]$.
3. Properties of $R[x]$ inherited from R – i.e., if R is commutative, if R has identity, if R is an integral domain ...
4. $\deg f(x)g(x) = \deg f(x) + \deg g(x)$ in $R[x]$ when R is an integral domain (Theorem 4.2).
5. The Division Algorithm in $F[x]$ (Theorem 4.4).
6. Definition of divides, factor, monic in $R[x]$.
7. Definition of the greatest common divisor of two polynomials and using the Euclidean Algorithm to write the gcd as a linear combination of those two polynomials.
8. A stronger definition for gcd (Corollary 4.6)
9. Properties of relatively prime polynomials (Theorem 4.7) (Note: Many of the theorems from \mathbb{Z} are true in $F[x]$ and would make good exam questions. For example, problems 28 and 29 of Section 1.2 could be adapted to $F[x]$.)
10. Units in $R[x]$ (Theorem 4.8).
11. Definition of associate, irreducible, and reducible (see Theorem 4.10 for reducible) in $F[x]$.
12. Properties of irreducible polynomials (Theorem 4.11).
13. Unique factorization into irreducibles (Theorem 4.13). (Note: many of our statements about primes can be adapted to $F[x]$, i.e., like problem 16 from Section 4.3.)
14. The Remainder Theorem (Theorem 4.14), the Factor Theorem (Theorem 4.15) and their consequences (Corollaries 4.16, 4.17, and 4.18)

15. Irreducibility in $\mathbb{Q}[x]$:

- (a) Rational Root Test (Theorem 4.20)
- (b) Factoring $f(x) \in \mathbb{Q}[x]$ with integer coefficients is the same as factoring in $\mathbb{Z}[x]$ (Theorem 4.22)
- (c) Eisenstein's Criterion (Theorem 4.23)
- (d) Irreducible in $\mathbb{Z}_p[x]$ for some prime p implies irreducible in $\mathbb{Q}[x]$ (Theorem 4.24).

16. Irreducibility in $\mathbb{R}[x]$ and $\mathbb{C}[x]$:

- (a) The Fundamental Theorem of Algebra (Theorem 4.25) and its consequences which characterize the irreducible polynomials in $\mathbb{C}[x]$ (Corollaries 4.26, 4.27)
- (b) Characterization of the irreducible polynomials in $\mathbb{R}[x]$ (Theorem 4.29) and its consequences (Corollary 4.30)

To prepare for this test, you should make sure that you have done each of the following:

- **Rewritten your class notes.** Anything that I asked you to finish, make sure you know how to finish it. You should understand all of the proofs and be able to apply the techniques used in class to similar problems.
- **Tried all of the homework problems,** even the ones that are not collected. Just because a problem was not collected does NOT mean that it is unimportant. Similar questions could appear on the exam.