

Study Guide for Exam 2

In what follows is a **brief** synopsis of what we have covered in Sections 2.2, 2.2, 3.1–3.3. Use this list a guide to help you make up your own study guide.

On the exam, you can expect several proofs, TRUE/FALSE questions, and give-an-example-of type questions (be able to give an example of anything defined below). The problems that have been assigned in class (but not necessarily collected) or very similar problems could appear on the exam; therefore it is **highly** recommended that you make every effort to complete those problems.

Exam 2 Topics:

1. Modular arithmetic, i.e., addition and multiplication in \mathbb{Z}_n .
2. Properties of modular arithmetic (Theorem 2.7).
3. Structure of \mathbb{Z}_p when p is prime (Theorem 2.8).
4. The number of solutions to $ax = b$ in \mathbb{Z}_n (Theorem 2.11 and its special cases: n prime (Corollary 2.9) and $(a, n) = 1$ (Corollary 2.10)).
5. Definition of a ring, commutative ring, ring with identity, commutative ring with identity, integral domain, field. Be able to give an example of each, i.e., give an example of a ring that is not commutative, a ring without identity, a commutative ring that is not an integral domain, an integral domain that is not a field, etc.
6. Definition of the ring $R \times S$ where R and S are rings (Theorem 3.1).
7. Showing a set S of a ring R is a subring using either Theorem 3.2 or Theorem 3.6.
8. Basic properties of rings: Theorems 3.3 (unique additive inverse), Theorem 3.4 (additive cancellation), Theorem 3.5.
9. Notation for additive inverse and subtraction.
10. Definition of a unit and notation for multiplicative inverse.
11. Property 12 implies property 11 (Theorem 3.9).
12. Know when multiplicative cancellation holds (Theorem 3.10) and when it does not.
13. Every finite integral domain is a field (Theorem 3.11).
14. Definition of zero divisors.
15. Definition of isomorphic rings and isomorphism.
16. Definition of homomorphism.

17. Properties of a homomorphism (Theorem 3.12).
18. Be able to show two rings are isomorphic or that two rings are not isomorphic.
19. Be able to give an example of two non-isomorphic rings, maybe with the same number of elements.
20. Definition of the image of a homomorphism and Corollary 3.13.

To prepare for this test, you should make sure that you have done each of the following:

- **Rewritten your class notes.** Anything that I asked you to finish, make sure you know how to finish it. You should understand all of the proofs and be able to apply the techniques used in class to similar problems.
- **Tried all of the homework problems,** even the ones that are not collected. Just because a problem was not collected does NOT mean that it is unimportant. Similar questions could appear on the exam.

Problems that were not assigned but could have been:

2.3: 12, 13

3.1: 19, 38

3.2: 2, 9, 10, 19, 22, 29

3.3: 18, 23