

Exam 4

106 points possible. 100 points maximum. Calculators must be used on some questions. To receive full credit, you **must** show all work!

1. (12 pts.)

(a) Express $\lim_{n \rightarrow \infty} \sum_{i=1}^n x_i \ln x_i \Delta x$ as a definite integral on the interval $[1, 4]$.

(b) Approximate the definite integral in part (a) using right Riemann sums with $n = 50$ rectangles.

(c) Do you think your answer in part (b) is an over or under estimate? Why or why not?

2. (10 pts.) Use Newton's Method to find one solution to the equation $6 \cos x = x$, correct to six decimal places. Write down the function you are applying Newton's Method to, its derivative, your initial guess, and **all** the iterates you find.

3. (12 pts.) Let f and g be functions such that $\int_1^4 f(x)dx = 7$, $\int_2^4 f(x)dx = 5$, and $\int_1^4 g(x)dx = 2$. Using this information, compute each of the following:

(a) $\int_1^4 4f(x)dx$

(b) $\int_1^4 (g(x) - f(x)) dx$

(c) $\int_1^2 -f(x)dx$

4. (10 pts.) A canister is dropped from a cliff 500 m above the ground. Attached to the canister is a parachute and the canister has been designed to withstand an impact velocity of 100m/s. If the parachute does not open, does the canister explode?

5. (10 pts.) Find the derivative of each function below, if it exists.

(a) $g(x) = \int_0^x \sqrt{1 + 2t} dt$

(b) $h(x) = \int_0^{e^x} \sin^3 t dt$

6. (15 pts.) Answer the following questions by circling **TRUE** or **FALSE**. BE CAREFUL in choosing your answer. No explanation necessary.

(a) **TRUE** or **FALSE** $\int_{-r}^r \sqrt{r^2 - x^2} dx = \frac{\pi r^2}{2}$

(b) **TRUE** or **FALSE** $\frac{d}{dx} \int_1^5 \ln t dt = \ln x$

(c) **TRUE** or **FALSE** Every function has an antiderivative.

(d) **TRUE** or **FALSE** $\lim_{x \rightarrow \infty} \frac{x^n}{e^x} = 0$ for every integer n .

(e) **TRUE** or **FALSE** If f and g are continuous on $[a, b]$ and $g(x) \neq 0$ on $[a, b]$, then

$$\int_a^b \frac{f(x)}{g(x)} dx = \frac{\int_a^b f(x) dx}{\int_a^b g(x) dx}$$

7. (10 pts.) Find the limits, if they exist. DO NOT USE YOUR CALCULATOR.

(a) $\lim_{x \rightarrow 0} \frac{e^x - 3x - e^{-x}}{x^2}$

(b) $\lim_{x \rightarrow 1} \frac{1}{x-1} - \frac{1}{\ln x}$

8. (15 pts.) Evaluate the following integrals BY HAND, if they exist. Show all work and use your calculator only as a check. Answers without any work will receive no credit.

(a) $\int_1^2 \frac{dx}{x^2}$

(b) $\int_0^2 (3e^x + 2 \sin x) dx$

(c) $\int x(1 + 2x^4) dx$

9. (12 pts.) The acceleration of an object moving in a straight line in another world far, far away is given by $a(t) = 2t + 3$ and its initial velocity $v(0)$ is -4 . Find the (a) displacement of the object and (b) the distance travelled by the particle over the time interval $[0, 3]$.