Math 145, Fall 2008
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Exam 2 Solutions

1. (a) **FALSE**
   (b) **FALSE**
   (c) **TRUE**
   (d) **TRUE**
   (e) **TRUE**

2.

$$\begin{aligned} y &= (\cos x)^x \\ \ln y &= x\ln(\cos x) \\ \frac{1}{y}\frac{dy}{dx} &= x\cdot\frac{1}{\cos x}\cdot -\sin x + \ln(\cos x) \\ \frac{dy}{dx} &= y(-x\tan x + \ln(\cos x)) \\ \frac{dy}{dx} &= (\cos x)^x(-x\tan x + \ln(\cos x)) \end{aligned}$$

3. (a) $f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ or $f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

   (b) $f'(a)$ is the slope of the tangent line to the graph of $y = f(x)$ at the point $(a, f(a))$; $f'(a)$ is the instantaneous rate of change of $y = f(x)$ with respect to x when $x = a$; $f'(a)$ is the instantaneous velocity of the position function $y = f(x)$ at time $x = a$.

   (c)

$$\begin{aligned} f'(a) &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \\ &= \lim_{h\to 0}\frac{\sqrt{3-5(a+h)}-\sqrt{3-5a}}{h} \\ &= \lim_{h\to 0}\frac{\sqrt{3-5(a+h)}-\sqrt{3-5a}}{h}\,\frac{\sqrt{3-5(a+h)}+\sqrt{3-5a}}{\sqrt{3-5(a+h)}+\sqrt{3-5a}} \\ &= \lim_{h\to 0}\frac{3-5(a+h)-(3-5a)}{h(\sqrt{3-5(a+h)}+\sqrt{3-5a})} \\ &= \lim_{h\to 0}\frac{-5h}{h(\sqrt{3-5(a+h)}+\sqrt{3-5a})} \\ &= \lim_{h\to 0}\frac{-5}{(\sqrt{3-5(a+h)}+\sqrt{3-5a})} \\ &= \frac{-5}{2\sqrt{3-5a}} \end{aligned}$$

4.

$$
\begin{aligned}
\frac{d}{dx}\left(x^2\cos y+\sin 2y\right. &= \left.xy\right)\\
x^2(-\sin y)\cdot y'+\cos y\cdot 2x+\cos 2y\cdot 2\cdot y' &= x\cdot y'+y\\
y'(-x^2\sin y+2\cos 2y-x) &= y-2x\cos y\\
y' &= \frac{y-2x\cos y}{-x^2\sin y+2\cos 2y-x}
\end{aligned}
$$

5. (a) $\dfrac{dy}{dx}=x\cdot\dfrac{1}{1+16x^2}\cdot 4+\tan^{-1}(4x)=\dfrac{4x}{1+16x^2}+\tan^{-1}(4x)$

(b) $y'=\dfrac{(1+\tan 2x)\sec 2x\tan 2x\cdot 2-\sec 2x\sec^2 2x\cdot 2}{(1+\tan 2x)^2}$

(c) $y'=\dfrac{1}{\sqrt{1-(1+x^3)}}\cdot\dfrac{1}{2}(1+x^3)^{-1/2}\cdot 3x^2$

(d) $\dfrac{dw}{dx}=2^{x\ln x}\cdot\ln 2\left(x\cdot\dfrac{1}{x}+\ln x\right)$

(e) $y'=-\sin(e^{\sqrt{\tan x}})\cdot e^{\sqrt{\tan x}}\cdot\frac{1}{2}(\tan x)^{-1/2}\sec^2 x$

6. (a) $g'(x)=x^2f'(x)+2xf(x)$

(b) $g'(x)=\dfrac{xf'(x)-f(x)}{x^2}$

(c) $g'(x)=e^{f(x)}f'(x)$

(d) $g'(x)=\dfrac{1}{f(x)}f'(x)$

(e) $g'(x)=f'(\tan x)\sec^2 x$

7. $\displaystyle\lim_{t\to 0}\frac{t^3}{\sin^3 2t}=\lim_{t\to 0}\left(\frac{2t}{\sin 2t}\right)^3\frac{1}{2^3}=1^3\cdot\frac{1}{2^3}=\frac{1}{8}$