

Answers to Exam 2

1. (a) **TRUE**
 (b) **FALSE**
 (c) **FALSE**
 (d) **FALSE**
 (e) **TRUE**
2. (a) The graph of $f(x) = -e^{-x}$.
 (b) The graph of a function $f(x) = e^x$.
3. (a) $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$
 (b) Slope of the tangent line to the graph $y = f(x)$ at $x = a$; instantaneous velocity at time $t = a$ where $y = f(t)$ is the position function.
 (c)

$$\begin{aligned}
 f'(a) &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{4(a+h)}{a+h+1} - \frac{4a}{a+1}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{4(a+h)(a+1) - 4a(a+h+1)}{h(a+h+1)(a+1)} \\
 &= \lim_{h \rightarrow 0} \frac{4a^2 + 4ah + 4a + 4h - (4a^2 + 4ah + 4a)}{h(a+h+1)(a+1)} \\
 &= \lim_{h \rightarrow 0} \frac{4h}{h(a+h+1)(a+1)} \\
 &= \lim_{h \rightarrow 0} \frac{4}{(a+h+1)(a+1)} \\
 &= \frac{4}{(a+1)^2}
 \end{aligned}$$

4.

$$\begin{aligned}
 \frac{d}{dx} (2x^2 + 6xy^2 - 7y^3) &= 1) \\
 4x + (6x) \cdot 2y \cdot y' + (y^2) \cdot 6 - 21y^2 \cdot y' &= 0 \\
 12xy \cdot y' - 21y^2 \cdot y' &= -4x - 6y^2 \\
 y'(12xy - 21y^2) &= -4x - 6y^2 \\
 y' &= \frac{-4x - 6y^2}{12xy - 21y^2}
 \end{aligned}$$

5.

$$\begin{aligned}\lim_{t \rightarrow 0} \frac{t}{\tan t} &= \lim_{t \rightarrow 0} \frac{t}{\left(\frac{\sin t}{\cos t}\right)} \\ &= \lim_{t \rightarrow 0} \frac{t}{\sin t} \cdot \cos t \\ &= \lim_{t \rightarrow 0} \frac{t}{\sin t} \cdot \lim_{t \rightarrow 0} \cos t \\ &= 1 \cdot 1 \\ &= 1\end{aligned}$$

6. (a) $y' = x \cdot \frac{1}{1 + (x + \pi)^2} \cdot 1 + \tan^{-1}(x + \pi) \cdot 1$

(b) $y' = e^{x \cos x} \cdot (x(-\sin x) + (\cos x) \cdot 1)$

(c) $y' = \frac{(1 + \sin x) \cdot 1 - x \cdot (\cos x)}{(1 + \sin x)^2}$

(d) $w' = 3(z + z^{-1} + 2z^{-2})^2 (1 - z^{-2} - 4z^{-3})$

(e) $s'(t) = \frac{(4t - 5)^5 \cdot 3(t^2 + 1)^2 \cdot 2t - (t^2 + 1)^3 \cdot 5(4t - 5)^4 \cdot 4}{(4t - 5)^{10}}$

7. (a) $f'(t)$ is the rate of change in temperature with respect to time in hours; the units are $^{\circ}\text{F}/\text{hrs}$.

(b) $f'(2) = -4$ means that the temperature is decreasing at a rate of 4 degrees when $t = 2$ hours.

(c) $f'(t)$ is always negative since the temperature will continue to decrease as time goes on.

8. (a) 47

(b) $-49/4$

(c) 30