Math 145, Fall 2008
H. Jordon

Exam 1 Solutions

1. (4 pts. each)

(a)

$$\begin{aligned}\lim_{x\to 3}\frac{\sqrt{x+6}-x}{x^3-3x^2} &= \lim_{x\to 3}\frac{\sqrt{x+6}-x}{x^2(x-3)}\frac{(\sqrt{x+6}+x)}{(\sqrt{x+6}+x)}\\ &= \lim_{x\to 3}\frac{(x+6)-x^2}{x^2(x-3)(\sqrt{x+6}+x)}\\ &= \lim_{x\to 3}\frac{-(x-3)(x+2)}{x^2(x-3)(\sqrt{x+6}+x)}\\ &= \lim_{x\to 3}\frac{-(x-2)}{x^2(\sqrt{x+6}+x)}\\ &= -\frac{5}{54}\end{aligned}$$

(b) $\lim_{r\to 9}\frac{\sqrt{r}}{(r-9)^4}=\infty$ since $\sqrt{r}\to 3$ and $(r-4)^4\to 0$ as $r\to 9$.

(c) $\lim_{x\to\infty}\frac{x+x^3+x^5}{1-x^2+x^4}=\lim_{x\to\infty}\frac{x^5}{x^4}=\lim_{x\to\infty}x=\infty$

(d)

$$\begin{aligned}\lim_{x\to 1}\left(\frac{1}{x-1}+\frac{1}{x^2-3x+2}\right) &= \lim_{x\to 1}\left(\frac{1}{x-1}+\frac{1}{(x-1)(x-2)}\right)\\ &= \lim_{x\to 1}\frac{(x-2)+1}{(x-1)(x-2)}\\ &= \lim_{x\to 1}\frac{x-1}{(x-1)(x-2)}\\ &= \lim_{x\to 1}\frac{1}{x-2}\\ &= -1\end{aligned}$$

(e) $\lim_{t\to 1^-}\ln(1-t)=-\infty$ since $\lim_{x\to 0^+}\ln x=-\infty$ and $1-t\to 0^+$ as $t\to 1^-$

(f) $\lim_{x\to\infty}e^{-x^2}=\lim_{x\to\infty}\frac{1}{e^{x^2}}=0$ since $e^{x^2}\to\infty$ as $x\to\infty$

2. (5 pts. each)

(a)

$$\begin{aligned} y &= e^{x-5} - 3 \\ y+3 &= e^{x-5} \\ \ln(y+3) &= x-5 \\ \ln(y+3)+5 &= x \end{aligned}$$

So, $f^{-1}(x) = \ln(x+3) + 5$.

(b)

$$\begin{aligned} y &= \ln x - \ln(x-1) \\ y &= \ln\left(\frac{x}{x-1}\right) \\ e^y &= \frac{x}{x-1} \\ e^y(x-1) &= x \\ e^y x - e^y &= x \\ e^y x - x &= e^y \\ x(e^y - 1) &= e^y \\ x &= \frac{e^y}{e^y - 1} \end{aligned}$$

So, $f^{-1}(x) = e^x/(e^x - 1)$.

3. (3 pts. each)

(a) **FALSE**

(b) **FALSE**

(c) **FALSE**

(d) **TRUE**

(e) **TRUE**

4. (8 pts.) Now $\lim_{x\to 1} 3x = 3$ and $\lim_{x\to 1} x^3 + 2 = 3$ and since $3x \le f(x) \le x^3 + 2$ for all $0 \le x \le 2$, by the Squeeze Theorem, $\lim_{x\to 1} f(x) = 3$.

5. (a) (10 pts.) Let $y = C \cdot a^t$. Then, $600 = C \cdot a^2$ and $75000 = C \cdot a^8$. So, $C = 600/a^2$. Using this fact, we obtain $75000 = (600/a^2)a^8$, or $125 = a^6$ so $a = \sqrt{5}$. Thus, $C = 120$ so that $y = 120 \cdot 5^{t/2}$.

(b) (5 pts.) $200,000 = 120 \cdot 5^{t/2}$. Solving for t, we get $t = 9.2188$ hrs.

6. (8 pts.) Let $s(t) = 2\sin\pi t + 3\cos\pi t$.

   (a) Define $m(t) = \dfrac{s(t) - s(2)}{t - 2} = \dfrac{2\sin\pi t + 3\cos\pi t - 3}{t - 2}$.

      i. $m(2.001) = 6.2684$
      ii. $m(2.0001) = 6.2817$
      iii. $m(1.999) = 6.298$
      iv. $m(1.9999) = 6.2847$

   (b) 6.3 cm/sec

7. (5 pts. each)

   (a) $y = \dfrac{2x^3}{|x(x+2)(x-1)|}$

   (b) $y = \ln(x+1)$

8. Let $g(x) = \begin{cases} x & \text{if } x \leq -1 \\ 1 - x^2 & \text{if } -1 < x < 1 \\ 2 & \text{if } x = 1 \\ x - 1 & \text{if } x > 1 \end{cases}$.

   (a) (1 pt. each)

      i. $\lim_{x\to -1^-} g(x) = -1$
      ii. $\lim_{x\to -1^+} g(x) = 0$
      iii. $\lim_{x\to -1} g(x)$ d.n.e.
      iv. $g(-1) = -1$
      v. $\lim_{x\to 1^-} g(x) = 0$
      vi. $\lim_{x\to 1^+} g(x) = 0$
      vii. $\lim_{x\to 1} g(x) = 0$
      viii. $g(1) = 2$

   (b) The function g is discontinuous at $x = -1$ since $\lim_{x\to -1} g(x)$ does not exist. The function g at $x = 1$ since $\lim_{x\to -1} g(x) \neq g(1)$.