

Name: _____

Math 145, Fall 2002

H. Gavlas

Exam 1, Part II

This part of the exam is worth 66 points and **calculators** may be used. To receive full credit, you **must** show all work!

1. (20 pts.) Answer the following questions by circling **TRUE** or **FALSE**. BE CAREFUL in choosing your answer. No explanation necessary.

(a) **TRUE** or **FALSE** If $x > 0$ and $a > 1$, then $e^x + e^a = e^{x+a}$.

(b) **TRUE** or **FALSE** The range of $y = \ln x^2$ is all positive real numbers.

(c) **TRUE** or **FALSE** Let a be a positive real number. If $a^x = y$, then $\log_a y = x$.

(d) **TRUE** or **FALSE** Let f be a function such that $f(-1) = 2$ and $f(1) = -1$. Then $f(x) = 0$ for some x in the interval $(-1, 1)$.

(e) **TRUE** or **FALSE** It is possible for $\lim_{x \rightarrow a} f(x) - g(x)$ to exist even if neither of $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist.

2. (16 pts.) Give an example of each of the following or state that no such example exists. No explanation necessary.

(a) The graph of a function f satisfying the following: $\lim_{x \rightarrow 2^+} f(x) = -1$, $\lim_{x \rightarrow 2^-} f(x) = 2$, $f(2) = 0$, $\lim_{x \rightarrow -1} f(x) = -1$, and $f(-1)$ is undefined.

(b) The equation of a function with a vertical asymptotes at $x = 2$ and $x = -1$ and a horizontal asymptote at $y = 3$.

- (c) The equation of an exponential function passing through the point $(0, 2)$.
- (d) The graph of a function that has a removable discontinuity at $x = -2$, a jump discontinuity at $x = 1$, and an infinite discontinuity at $x = 4$.
3. (8 pts.) An object is fired directly upwards. Its height $s(t)$ in feet above the ground after t seconds is given by $s(t) = 144t - 16t^2$.
- (a) Find the average velocity over the following time intervals:
- $[3, 3.01]$
 - $[2.99, 3]$
- (b) From your work in the previous question, estimate the instantaneous velocity of the object after 3 seconds.
4. (8 pts.) Use the Intermediate Value Theorem to show that the equation $x^5 - x^2 + 2x + 3 = 0$ has at least one real root.

5. (14 pts.) The analysis of tooth shrinkage by scholars at the University of Michigan indicates that human tooth size is continuing to decrease and that the evolutionary process had not come to a halt as many scientists contend. In northern Europeans, for example, tooth size reduction now has a rate of 1% per 1000 years. (This means that the tooth size will be 99% of what it is today in 1000 years.)

(a) Suppose that tooth size today is Q_0 . Express the tooth size Q as a function of time t .

(b) In about how many years will human teeth be 90% of their present size?

(c) What will be our descendants' tooth size 20,000 years from now (as a percentage of our present tooth size)?