MATHEMATICAL ABUNDANCE - DESIGNS, GRAPHS, NUMBER THEORY

PLENARY TALKS

Perfect Matchings in Bipartite Graphs: Properties and Structure
Richard A. Brualdi, University of Wisconsin **[F 9:00 357]**

Perfect matchings in bipartite graphs have been extensively studied. If two graphs are isomorphic, then their perfect matchings are in a natural bijection. But what about the converse? Of course, it is not true, but what is true? Perfect matchings are also connected with the polytope of doubly stochastic matrices and its faces (a face can be viewed as a bipartite graph). This gives a geometric structure to the perfect matchings of bipartite graphs.

What can we still learn from Euler about partitions
George Andrews, Penn State University **[F 2:00 357]**

Euler devotes chapter 16 of his book, Introduction to the Analysis of the Infinite Vol. 1 (John Blanton, translator), to the partitions of numbers. I wrote an article for the Bulletin of the American Mathematical Society on this chapter. After surveying Euler's achievements, I pointed to a number of new partitions theorems inspired by Euler's ideas. In this talk I will elaborate presenting further new results that have arisen from my first observations.

Ramanujan's Series for 1/pi
Bruce C. Berndt, University of Illinois-Urbana **[S 9:00 357]**

In his famous paper, Modular Equations and Approximations to Pi, Ramanujan recorded 17 hypergeometric-like series representations for 1/pi. These were not completely proved until 1987 when Jonathan and Peter Borwein found proofs. In the past 20 years, several authors have found new hypergeometric-like series for 1/pi. In particular, in the past two years, Heng Huat Chan, Nayandeep Deka Baruah, and the speaker have returned to Ramanujan's paper and used his ideas, which are based on Eisenstein series, more so than previous authors to establish proofs of most of Ramanujan's formulas and to discover many new such formulas as well. A historical survey of attempts to prove Ramanujan's formulas will be given, emphasizing the contributions of S. Chowla, R. William Gosper, Jr., Jonathan and Peter Borwein, David and Gregory Chudnovsky, Heng Huat Chan, Nayandeep Baruah, and others. Ramanujan's ideas arising from Eisenstein series will be explained.

Discourse on three combinatorial diseases
Alex Rosa, McMaster University **[S 2:00 357]**

How does one diagnose a combinatorial disease? It is usually highly infectious, and the problem has an elementary formulation which more often than not is understandable to a good high school student. Of the combinatorial diseases that have been identified starting with Harary's paper some forty years ago, only one has been cured so far; but in some other cases we understand now better why a "cure" as it was originally envisaged is not likely to be forthcoming. In my talk, I would like to expand on some of my favourite combinatorial diseases, in the hope that this will spur renewed, and hopefully more successful, attempts at curing at least some of them. These are: the labelling disease, the Erdös-Faber-Lovász disease, and the Buratti problem.

CONTRIBUTED TALKS

(In alphabetical order of last names of speakers)

Sentry Selection
Paul Balister, The University of Memphis

[F 3:25 354]

Suppose we have a collection of sensors in a large region, each of which can detect events within a disk of radius 1. We wish to devise a schedule so that each sensor can sleep for much of the time, while making sure that the whole region is covered by the sensors that are awake. A natural way of doing this is to partition the sensors into k subsets, each subset of sensors covering the whole region. Then in time slot t we activate all the sensors in subset ($t \bmod k$). If this is possible we say the sensors are k-partitionable. An obvious necessary condition is that each point in the region is covered by at least k sensors (k-coverage), but this is not in general sufficient. We show that for random deployments of sensors k-coverage usually implies k-partitionability, and identify the most likely obstructions to k-partitionability when this fails. This leads to some natural unsolved problems involving k-partitionability of (deterministic) configurations of disks. This is joint work with B. Bollobas, A. Sarkar, and M. Walters.

A flexagon duality
Frank R. Bernhart, Currently visiting University of Illinois-Urbana

[F 4:55 353]

We bring together the theory of the standard flexagon and the study of graphs obtained by adding diagonals to convex polygons. A new duality emerges, involving two infinite families of models, and two associated methods of flexing, each of which is guaranteed to generate a full traverse or tour of the network of faces. The family of "street" flexagons and the Tuckerman traverse have been known since Arthur Stone invented the flexagon in 1939. The other family and the other traverse appear not to have been noticed before now. The methods used involve the class of triangulated polygons, a checkerboard shading, and a new involution on this class we call the "twist". This transformation has the added bonus of unscrambling certain features, so that we can simply read out the shape and labeling of the unfolded strip from which a working model is made.

Finding "Bunch Permutations"
G.S. Bloom, The City College / CUNY

[F 11:25 353]

Call a set $B = \{x_i : 0 \le i \le n, x_i \ge 2\}$ a *bunch partition* of $M = \sum_{i=1}^{n} x_i$. We consider each permutation of B to be the input for a set of instructions to create a square $(1 + n + M)$-Boolean matrix. If a particular permutation is "good" we can form the matrix, and the permutation is called a *bunch permutation.* If the permutation is "bad" (violates conditions in the instruction set), we try another permutation of B to see if it is good. We have experimentally determined that for $M \le 57$, every bunch partition of B has a good permutation, and we have found that we don't usually have to search very long to find a good one. What we don't know is how to translate the algorithm for creating an appropriate matrix starting from the permutation into conditions on the permutation that tell us if we are looking at a good or bad one. And since we conjecture that every bunch partition has a good permutation, we would like to know how to formulate any one bunch permutation directly from its underlying partition without going through the process of creating the matrix. Motivation: If our conjecture is correct, all banana trees are graceful.

Pincherle type theorems for sequential closures of continued fractions
Douglas Bowman, Northern Illinois University

[S 3:25 353]

One of the most powerful methods for establishing the limits of convergent continued fractions is Pincherle's theorem. The approach via Pincherle's theorem is to compute a linearly independent solution set of the adjoint recurrence for a continued fraction. When one of the solutions of the recurrence is minimal, then the limit of the continued fraction is simply the ratio of the first two terms of the miniamal solution. In recent research (with James McLaughlin) I have described a new convergence theory for continued fractions which allow one to compute the sets of limits of all subsequences of approximants for continued fractions, even when the continued fraction diverges. The weakness of the method, however, is that to comput these sets of limits (called sequential closures) one needs asymptotic information about the classical numerators and denominators at infinity. Here I present an analogue of the Pincherle theorem which eliminates this requirement and thus enables the computation of the sequential closure from knowledge of the solutions of the adjoint equation instead. As a consequence, I am able to compute sequential closures for continued fractions with more free parameters. Examples will be presented.

Base inversion in certain classes of q-continued fractions
Kristen Campbell, Northern Illinois University

[F 11:55 353]

For the purposes of this talk we define a q-continued fraction as one in which the nth partial numerator and denominator are each fixed polynomials in q^n. (Here q is a complex number of modulus less than 1.) What can be said about such continued fractions under the transformation $q \mapsto aq^{-1}$, where a is a fixed complex constant? We first present a general result showing that the cardinality of the sequential closures for such continued fractions is easy to compute in general. (The sequential closure is defined to be the set of limits of convergent subsequences.) Next we examine a couple of examples in detail and consider relations between the limits before and after such a transformation. This is joint work with Douglas Bowman, Northern Illinois University.

Super edge-graceful labelings of paths
Sylwia Cichacz, University of Minnesota-Duluth

[S 3:55 354]

A graph $G(V,E)$ of order $|V| = p$ and size $|E| = q$ is called super edge-graceful if there is a bijection f from E to $\{0, \pm 1, \pm 2, \ldots, \pm \frac{q-1}{2}\}$ when q is odd and from E to $\{\pm 1, \pm 2, \ldots, \pm \frac{q}{2}\}$ when q is even such that the induced vertex labeling f^* defined by $f^*(x) = \sum_{xy \in E(G)} f(xy)$ over all edges xy is a bijection from V to $\{0, \pm 1, \pm 2 \ldots, \pm \frac{p-1}{2}\}$ when p is odd and from V to $\{\pm 1, \pm 2, \ldots, \pm \frac{p}{2}\}$ when p is even. We prove that all paths P_n except P_2 and P_4 are super edge-graceful.

Proof of the Bollobás Conjecture for Embedding Bounded Degree Trees
Béla Csaba, Western Kentucky University

[S 10:25 354]

An old conjecture of Bollobás is the following: If G is a simple graph on n vertices with $\delta(G) \geq (1/2 + \eta)n$ for some $\eta > 0$ and T is a bounded degree tree on n vertices, then $T \subset G$ if n is large enough. Recently we showed a stronger version: $\delta(G) \geq n/2 + c \log n$ is sufficient to guarantee that T is a spanning subgraph of G if n is larger than a threshold. This is a joint work with Endre Szemerédi and Ian Levitt, Rutgers University.

Cyclic decompositions of complete graphs into $K_{m,n} + e$: the missing case
Dalibor Froncek, University of Minnesota-Duluth

[S 3:25 354]

A graph G is called *almost bipartite* if there exists an edge $e \in E(G)$ such that $G - e$ is bipartite. In 2004, A. Blinco, S. El-Zanati, and C. Vanden Eynden defined a new type of labeling, called γ-labeling, whose existence for a graph G with p edges guarantees a cyclic G-decomposition of K_{2px+1} for any positive integer x. In a very recent preprint, S. El-Zanati, W. O'Hanlon, and E. Spicer used the labeling to show that if $n > 2$ and $K_{m,n} + e$ arises from $K_{m,n}$ by adding an edge into the partite set of size n, then $K_{m,n} + e$ has a γ-labeling and therefore cyclically decomposes K_{2kx+1}, where $k = nm + 1$, for any positive integer x. They also showed that for $n = 2$ no γ-labeling exists. In the conclusion of their paper, however, they expressed a belief that a cyclic decomposition still exists. In this talk, we are going to confirm their belief.

Complete H-decompositions
Zoltán Füredi, University of Illinois-Urbana

[F 10:25 354]

Let H be a simple graph. An *H-packing* of order n is a set $\mathcal{P} := \{H_1, H_2, \ldots, H_m\}$ of edge disjoint copies of H whose union forms a graph with n vertices. If this graph is the *complete graph* K_n, then $\mathcal{P}$ is called a *perfect* H-packing on n vertices, or following the terminology of design theory, it is called an *H-design* of order n. The case $H = K_k$ is equivalent to the existence of Steiner systems $S(n, k, 2)$. Let $f(n; H)$ be the smallest integer t such that, any H-packing on n vertices can be extended to a perfect H-packing on at most $n + t$ vertices. The existence of $f(n; H)$ follows from a result of Wilson 1975. There are many explicit constructions to provide linear upper bounds $f(n, H) < c_H n$ by Hoffman, Lindner, Rodger, and Stinson, by Jenkins, by Küçükçifçi, Lindner, and Rodger. Bryant, Khodkar, and El-Zanati gave explicit upper bounds (linear in n) for an infinite class of bipartite H. Here we give an asymptotic that $f(n, C_4) = (1 + o(1))\sqrt{n}$. Most of this talk is based on work with Hilton, Lehel and Lindner.

Two Identities for Squares of the Rogers-Ramanujan Functions and Applications
Chadwick Gugg, University of Illinois-Urbana

[S 10:55 353]

In his notebooks, Ramanujan recorded 40 beautiful modular relations for the Rogers-Ramanujan functions. He also recorded modular relations for the Rogers-Ramanujan continued fraction, $R(q)$. In particular, he defined the parameters $k := R(q)R^2(q^2), \mu := R(q)R(q^4)$, and $\nu := R^2(q^{1/2})R(q)/R(q^2)$, and gave a number of elegant relations for these parameters. Starting from identities for the Rogers-Ramanujan functions, we give new proofs of some of these results.

Set systems and graph representations
Peter Hamburger, Western Kentucky University

[F 11:25 354]

In this talk we will explore the connections between set systems and representations of graphs. We will introduce a new one that we call a Kneser representation of a graph. Then we will compare it with known ones such as the Prague representation, the modulo n representation for a positive integer n, and orthogonal Latin square representation. We also explore the connections with known coloring results of graphs. This is joint work with Attila Por, Western Kentucky University, and Matt Walsh, Indiana University Purdue University Fort Wayne, IPFW.

Computer Methods for Finding Graph Decompositions
Stephen Hartke, University of Nebraska-Lincoln

[S 4:25 354]

We will discuss computer methods for finding graph decompositions of a complete graph into several copies of a regular graph. Such decompositions have a high amount of symmetry, and thus require techniques to reduce the search space. We will discuss our search and the results obtained so far.

Multiclique partitions of multigraphs,
and regular satisfiability problems with non-boolean variables
Matthew Henderson, Western Kentucky University

[S 4:55 354]

In this talk we report on investigations into biclique partitioning of complete graphs generalised to multiclique partitioning of complete multigraphs. Our emphasis is on the connection between these two graph-theoretic problems and the conflict structure of boolean and non-boolean conjunctive normal forms (viewed as hypergraphs with signs). We exploit this connection to discuss some open problems from graph theory (the Alon-Saks conjecture) and design theory (the existence of associative block designs). This is joint work with Oliver Kullmann, Swansea University.

A resolvable decomposition
Sarah Holliday, University of Tennessee-Martin

[S 4:55 353]

The problem of resolvably decomposing complete graphs, bipartite graphs and multipartite graphs (sometimes with a one-factor removed) is referred to as Oberwolfach problem. I'll give some old results, some new results, and a neat little lemma about decomposing bipartite graphs.

Ramanujan's "alternative theories" via differential equations for Eisenstein series
Tim Huber, Iowa State University

[S 10:25 353]

Ramanujan's representations for Eisenstein series in terms of hypergeometric series are extremely useful in number theory, and, in particular, in Ramanujan's own work. This lecture will demonstrate the derivation of formulae for Eisenstein series included in Ramanujan's theory of elliptic functions to alternative bases. In contrast to the usual derivations, where an analogue of the Jacobi inversion formula is used, these parameterizations will be constructed directly from the differential equations satisfied by the classical Eisenstein series. The corresponding elementary techniques lead to results outside the scope of Ramanujan's alternative theories.

What is a space-time coordinate system on a graph?
Lucian Ionescu, Illinois State University

[S 4:25 353]

The Min-Cut-Max-Flow Theorem regards the flow of a conserved quantity on a graph. A comparison with the differential equation framework raises an important question: is there a "good" notion of a space-time coordinate system, with some reasonable equation of dynamics associated to it? A model is proposed, but suggestions from the audience are most welcome.

The role of symmetry in cage constructions
Robert Jajcay, Indiana State University

[F 4:55 354]

Recently, a significant number of papers has appeared containing new constructions of infinite families of small regular graphs of fixed girth that take the basic idea from the original well-known highly symmetric constructions, but result in families with fewer automorphisms. The aim of the talk is to address the importance and/or limitations of symmetry in cage constructions. This is joint work with Geoff Exoo.

Bounds for signed edge domination numbers in graphs
Abdollah Khodkar, University of West Georgia

[S 10:55 354]

The closed neighborhood $N_G[e]$ of an edge e in a graph G is the set consisting of e and of all edges having a common end-vertex with e. Let f be a function on $E(G)$, the edge set of G, into the set $\{-1, 1\}$. If $\sum_{x \in N[e]} f(x) \geq 1$ for each $e \in E(G)$, then f is called a signed edge dominating function of G. The minimum of the values of $\sum_{x \in E(G)} f(x)$, taken over every signed edge dominating function f of G, is called the signed edge domination number of G and is denoted by $\gamma'_s(G)$. It has been conjectured that $\gamma'_s(G) \leq n - 1$ for every simple graph G of order n. In this talk we see that this conjecture is true for Eulerian simple graphs, simple graphs with all vertices of odd degree and hence regular graphs. As a result we prove that for any simple graph G of order n, $\gamma'_s(G) \leq \lceil \frac{3n}{2} \rceil$. This improves the previous upper bound of $\lfloor \frac{11n}{6} - 1 \rfloor$. This is joint work with H. Karami and S.M. Sheikholeslami, Department of Mathematics, Azarbaijan University of Tarbiat Moallem Tabriz, Iran.

The Overpartition Function Modulo Small Powers of 2
Byungchan Kim, University of Illinois-Urbana

[F 11:55 354]

We will prove that the overpartition function $\overline{p}(n)$ is divisible by 128 for almost all integers n. We will also completely determine arithmetic properties of the overpartition function modulo 8.

Partitions and Edge Colorings of Multigraphs
A. Kostochka, University of Illinois-Urbana

[S 11:25 354]

Let $\chi(G)$ and $\omega(G)$ denote the chromatic number and the clique number of a graph G, respectively. Erdős and Lovász conjectured in 1968 that for every graph G with $\chi(G) > \omega(G)$ and any two integers $s, t \geq 2$ with $s + t = \chi(G) + 1$, there is a partition $V(G) = S \cup T$ such that $\chi(G[S]) \geq s$ and $\chi(G[T]) \geq t$. Except for a few cases, this conjecture is unsolved. We prove the conjecture for line graphs of multigraphs. The talk is based on joint work with M. Stiebitz.

Classifying distance-regular graphs
Michael Lang, Bradley University

[F 4:25 353]

Distance-regular graphs appear in a number of fields, including coding theory and knot theory. We discuss techniques and partial results for the problem of classifying distance-regular graphs.

Graph-theoretic generalization of the secretary problem: the directed path case
Michal Morayne, Wroclaw University of Technology

[Cancelled]

There is a natural generalization of the best choice problem that has been considered in many different variants for linear and partial orders. Namely, we can consider its analogue for graphs. The vertices ordered randomly into a permutation π (all permutations are equiprobable) appear one by one at the observer who at the moment t can observe the induced subgraph on the t vertices $\pi(1), \ldots, \pi(t)$ which have already appeared. The aim of the observer is to choose at time t a vertex $\pi(t)$ maximizing the probability that it has some property in the whole graph. We solve this problem for the directed path case. The aim here is to choose the vertex with no outgoing edges (the one on the "top") - the analogue of the maximal element for the linear order. An optimal stopping rule will be described as well as the probability of success and its asymptotic behavior. This is joint work with Grzegorz Kubicki.

Subsets in convex position of a planar point set and cliques in a related graph
Walter Morris, George Mason University

[F 4:25 354]

To a nondegenerate set of n points in the plane, one can associate a graph that has less than n^2 vertices and has the property that k-cliques in the graph correspond to vertex sets of convex k-gons in the point set. We prove an upper bound of 2^{k-1} on the size of a planar point set for which the graph has chromatic number k, matching the bound conjectured by Szekeres for the clique number.

On jumping densities of hypergraphs
Yuejian Peng, Indiana State University

[S 3:55 353]

A number $\alpha \in [0, 1)$ is a jump for an integer $r \geq 2$ if there exists a constant $c > 0$ such that for any family $\mathcal{F}$ of r-uniform graphs, if the Turán density of $\mathcal{F}$ is greater than α, then the Turán density of $\mathcal{F}$ is at least $\alpha + c$. A fundamental result in extremal graph theory due to Erdős and Stone implies that every number in $[0, 1)$ is a jump for $r = 2$. Erdős also showed that every number in $[0, r!/r^r)$ is a jump for $r \geq 3$. However, not every number in $[0, 1)$ is a jump for $r \geq 3$. In fact, Frankl and Rödl showed the existence of non-jumps for $r \geq 3$. In this talk, we describe more recent results for $r \geq 3$.

Construction of a Small(?) Regular Graph of Girth 5 and Degree 19
Allen Schwenk, Western Michigan University

[F 10:55 354]

The smallest orders for regular graphs of girth 5 are known for each degree up to 7. For higher degree these are harder to find. We give an algebra-based construction for degree 19 that is, so far as we know, the smallest constructed to date.

Divisibility by 3 of the number of matchings of a family of planar graphs
Naeem Sheikh, University of Illinois-Urbana

[F 3:55 353]

Let G_n be the planar graph obtained by taking the triangular lattice graph with $n+1$ rows of vertices (n rows of triangles) and then in each triangular face "facing up", put an extra vertex and make it adjacent to the 3 vertices of the enclosing face. Propp conjectured that for G_n, the number of (perfect) matchings is divisible by 3. In joint work with Stephen Hartke and Kyung-won Hwang, we have shown that in fact $3^{(n+1)/2}$ divides the number of matchings of G_n. We have also proved divisibility by an extra factor of 3 when n is 0 mod 3.

One and one third orthogonal Latin squares
W. D. Wallis, Southern Illinois University

[F 3:55 354]

It is obvious that the solution of a sudoku puzzle involves construction of a special kind of Latin square. We examine some statistical applications of this.

Zumkeller numbers
Matt Walsh, Indiana-Purdue University

[F 10:55 353]

A Zumkeller number is a positive integer with factors that can be partitioned into two equal-summed parts; this represents a generalization of perfect numbers, which admit a unique partition of this form. In this talk I shall discuss constructive methods for demonstrating several families of Zumkeller numbers and examine some variations on the theme.

Repetition number of graphs
Douglas B. West, University of Illinois-Urbana

[S 11:55 354]

Every n-vertex graph has two vertices with the same degree (if $n \geq 2$). In general, let $\text{rep}(G)$ be the maximum multiplicity of a vertex degree in G. An easy counting argument yields $\text{rep}(G) \geq n/(2d - 2s + 1)$, where d is the average degree and s is the minimum degree of G. Equality can hold when $2d$ is an integer, and the bound is approximately sharp in general, even when G is restricted to be a tree, maximal outerplanar graph, planar triangulation, or claw-free graph. Among large claw-free graphs, repetition number 2 is achievable, but if G is an n-vertex line graph, then $\text{rep}(G) \geq \frac{1}{4}n^{1/3}$. Among line graphs of trees, the minimum repetition number is $\Theta(n^{1/2})$. For line graphs of maximal outerplanar graphs, trees with perfect matchings, or triangulations with 2-factors, the lower bound is linear. This is joint work with Yair Caro, Univ. Haifa-Oranim.